Answers
To the Odd-Numbered Problems

Section 1.1

1. $1 + 2i$

3. $-2/5$

5. $2 + 2i\sqrt{3}$

7. $4e^{\pi i}$

9. $5\sqrt{2}e^{3\pi i/4}$

11. $2e^{2\pi i/3}$

Section 1.2

1.

$$\pm\sqrt{2}, \qquad \pm\sqrt{2}\left[\frac{1}{2} + \frac{\sqrt{3}i}{2}\right], \qquad \pm\sqrt{2}\left[-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right]$$

3.

$$i, \qquad -\frac{\sqrt{3}}{2} - \frac{i}{2}, \qquad z_2 = \frac{\sqrt{3}}{2} - \frac{i}{2}$$

5.

$$w_1 = \frac{1}{\sqrt{2}}\left[-\sqrt{\sqrt{a^2+b^2}+a} + i\sqrt{\sqrt{a^2+b^2}+a}\right], \qquad w_2 = -w_1.$$

7. $z_{1,2} = \pm(1+i); z_{3,4} = \pm 2(1-i)$

Section 1.3

1. $u = 2 - y, \ v = x$

3. $u = x^3 - 3xy^2, \ v = 3x^2y - y^3$

5. $f'(z) = 3z(1+z^2)^{1/2}$

7. $f'(z) = 2(1+4i)z - 3$

9. $f'(z) = -3i(iz-1)^{-4}$

11. 1/6

13. $v(x,y) = 2xy + \text{constant}$

15. $v(x,y) = x\sin(x)e^{-y} + ye^{-y}\cos(x) + \text{constant.}$

Section 1.4

1. 0

3. $2i$

5. $14/15 - i/3$

Section 1.5

1. $(e^{-2} - e^{-4})/2$

3. $\pi/2$

Section 1.6

1. $\pi i/32$

3. $\pi i/2$

5. $-2\pi i$

7. $2\pi i$

9. -6π

Section 1.7

1.
$$\sum_{n=0}^{\infty}(n+1)z^n.$$

3.
$$f(z) = z^{10} - z^9 + \frac{z^8}{2} - \frac{z^7}{6} + \cdots - \frac{1}{11!z} + \cdots$$
We have an essential singularity and the residue equals $-1/11!$

5.
$$f(z) = \frac{1}{2!} + \frac{z^2}{4!} + \frac{z^4}{6!} + \cdots$$
We have a removable singularity where the value of the residue equals zero.

7.

$$f(z) = -\frac{2}{z} - 2 - \frac{7z}{6} - \frac{z^2}{2} - \cdots$$

We have a simple pole and the residue equals −2.

9.

$$f(z) = \frac{1}{2}\frac{1}{z-2} - \frac{1}{4} + \frac{z-2}{8} - \cdots$$

We have a simple pole and the residue equals 1/2.

Section 1.8

1. $-3\pi i/4$ 3. $-2\pi i$. 5. $2\pi i$ 7. $2\pi i$

Section 2.1

1.

$$f(t) = \frac{1}{2} - \frac{2}{\pi}\sum_{m=1}^{\infty} \frac{\sin[(2m-1)t]}{2m-1}$$

3.

$$f(t) = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2\pi}\cos(nt) + \frac{1-2(-1)^n}{n}\sin(nt)$$

5.

$$f(t) = \frac{4}{\pi^2}\sum_{m=1}^{\infty} \frac{\cos[(2m-1)\pi t]}{(2m-1)^2}$$

7.

$$f(t) = \frac{1}{\pi} + \frac{1}{2}\sin(t) - \frac{2}{\pi}\sum_{m=1}^{\infty} \frac{\cos(2mt)}{4m^2-1}$$

9.

$$f(t) = -\frac{4}{\pi^2}\sum_{m=1}^{\infty} \frac{(-1)^m}{(2m-1)^2}\sin[(2m-1)\pi t]$$

11.

$$f(t) = \frac{a}{2} - \frac{4a}{\pi^2}\sum_{m=1}^{\infty} \frac{1}{(2m-1)^2}\cos\left[\frac{(2m-1)\pi t}{a}\right] - \frac{2a}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\sin\left(\frac{n\pi t}{a}\right)$$

13.

$$f(t) = \frac{L}{2\pi}\sin\left(\frac{\pi t}{L}\right) - \frac{2L}{\pi}\sum_{n=2}^{\infty}\frac{(-1)^n n}{n^2-1}\sin\left(\frac{n\pi t}{L}\right)$$

15.

$$f(t) = \frac{4a\cosh(a\pi/2)}{\pi}\sum_{m=1}^{\infty}\frac{\cos[(2m-1)t]}{a^2+(2m-1)^2}.$$

Section 2.3

1.

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{\cos[(2m-1)x]}{(2m-1)^2}$$

$$f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin(nx)}{n}$$

3.

$$f(x) = \frac{1}{4} - \frac{2}{\pi^2}\sum_{m=1}^{\infty}\frac{\cos[2(2m-1)\pi x]}{(2m-1)^2}$$

$$f(x) = \frac{4}{\pi^2}\sum_{m=1}^{\infty}\frac{(-1)^{m+1}\sin[(2m-1)\pi x]}{(2m-1)^2}$$

5.

$$f(x) = \frac{3}{4} + \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos^2\left(\frac{n\pi}{4}\right)\cos\left(\frac{n\pi x}{2}\right)$$

$$f(x) = \frac{4}{\pi^2}\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{(2m-1)^2}\sin\left[\frac{(2m-1)\pi x}{2}\right] - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\left(\frac{n\pi x}{2}\right)$$

7.

$$f(x) = \frac{3}{4} + \frac{1}{\pi}\sum_{m=1}^{\infty}\frac{(-1)^m}{2m-1}\cos\left[\frac{(2m-1)\pi x}{a}\right]$$

$$f(x) = \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1+\cos(n\pi/2)-2(-1)^n}{n}\sin\left(\frac{n\pi x}{a}\right)$$

9.

$$f(x) = \frac{3a}{8} + \frac{2a}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(n\pi/2)-1}{n^2}\cos\left(\frac{n\pi x}{a}\right)$$

$$f(x) = \frac{a}{\pi}\sum_{n=1}^{\infty}\left[\frac{2}{n^2\pi}\sin\left(\frac{n\pi}{2}\right) - \frac{(-1)^n}{n}\right]\sin\left(\frac{n\pi x}{a}\right)$$

11.

$$f(x) = \frac{1}{2} + \frac{2}{\pi}\sum_{m=1}^{\infty}\frac{(-1)^m}{m}\sin\left(\frac{m\pi}{2}\right)\cos\left(\frac{2m\pi x}{a}\right)$$

$$f(x) = \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{2m-1}\sin\left[\frac{(2m-1)\pi}{4}\right]\sin\left[\frac{(2m-1)\pi x}{a}\right]$$

13.

$$f(x) = \frac{e^{ak}-1}{ak} + 2ka\sum_{n=1}^{\infty}\frac{(-1)^n e^{ka}-1}{k^2a^2+n^2\pi^2}\cos\left(\frac{n\pi x}{a}\right)$$

$$f(x) = -2\pi\sum_{n=1}^{\infty}\frac{n[(-1)^n e^{ka}-1]}{k^2a^2+n^2\pi^2}\sin\left(\frac{n\pi x}{a}\right)$$

Section 2.4

1.

$$f(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin[(2n-1)t]}{2n-1}$$

$$f(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos[(2n-1)t-\pi/2]}{2n-1}$$

3.

$$f(t) = 2\sum_{n=1}^{\infty}\frac{1}{n}\cos\left[nt + (-1)^n\frac{\pi}{2}\right]$$

$$f(t) = 2\sum_{n=1}^{\infty}\frac{1}{n}\sin\left\{nt + [1+(-1)^n]\frac{\pi}{2}\right\}$$

Section 2.5

1.

$$f(t) = \frac{\pi}{2} - \frac{2}{\pi} \sum_{m=-\infty}^{\infty} \frac{e^{i(2m-1)t}}{(2m-1)^2}$$

3.

$$f(t) = 1 + \frac{i}{\pi} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{e^{n\pi i t}}{n}.$$

5.

$$f(t) = \frac{1}{2} - \frac{i}{\pi} \sum_{m=-\infty}^{\infty} \frac{e^{2(2m-1)it}}{2m-1}.$$

Section 2.6

1.

$$y(t) = A\cosh(t) + B\sinh(t) - \frac{1}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin[(2n-1)t]}{(2n-1)+(2n-1)^3}$$

3.

$$y(t) = Ae^{2t} + Be^{t} + \frac{1}{4} + \frac{6}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)t]}{[2-(2n-1)^2]^2 + 9(2n-1)^2}$$
$$+ \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{[2-(2n-1)^2]\sin[(2n-1)t]}{(2n-1)\{[2-(2n-1)^2]^2 + 9(2n-1)^2\}}$$

5.

$$y_p(t) = \frac{\pi}{8} - \frac{2}{\pi} \sum_{n=-\infty}^{\infty} \frac{e^{i(2n-1)t}}{(2n-1)^2[4-(2n-1)^2]}$$

7.

$$q(t) = \sum_{n=-\infty}^{\infty} \frac{\omega^2 \varphi_n}{(in\omega_0)^2 + 2i\alpha n\omega_0 + \omega^2} e^{in\omega_0 t}$$

Section 2.7

1. $x(t) = \frac{3}{2} - \cos(\pi x/2) - \sin(\pi x/2) - \frac{1}{2}\cos(\pi x)$

Section 3.3

1. $\pi e^{-|\omega/a|}/|a|$

Section 3.4

1. $-t/(1+t^2)^2$

3. $f(t) = \frac{1}{2}e^{-t}H(t) + \frac{1}{2}e^{t}H(-t)$

5. $f(t) = e^{-t}H(t) - e^{-t/2}H(t) + \frac{1}{2}te^{-t/2}H(t)$

7.
$$f(t) = \frac{i}{2}\operatorname{sgn}(t)e^{-a|t|}, \quad \text{where} \quad \operatorname{sgn}(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0. \end{cases}$$

9.
$$f(t) = \frac{1}{4a}(1 - a|t|)e^{-a|t|}$$

11.
$$f(t) = \frac{(-1)^{n+1}}{(2n+1)!}t^{2n+1}e^{-at}H(t)$$

13.
$$f(t) = \begin{cases} e^{2t}, & t > 0 \\ e^{-t}, & t < 0. \end{cases}$$

Section 3.6

1.
$$y(t) = [(t-1)e^{-t} + e^{-2t}]H(t)$$

3.
$$y(t) = \begin{cases} \frac{1}{9}e^{-t}, & t > 0 \\ \frac{1}{9}e^{2t} - \frac{1}{3}te^{2t}, & t < 0. \end{cases}$$

Section 4.1

1. $F(s) = s/(s^2 - a^2)$

3. $F(s) = 1/s + 2/s^2 + 2/s^3$

5. $F(s) = \left[1 - e^{-2(s-1)}\right]/(s-1)$

7. $F(s) = 2/(s^2+1) - s/(s^2+4) + \cos(3)/s - 1/s^2$

9. $f(t) = e^{-3t}$

11. $f(t) = \frac{1}{3}\sin(3t)$

13. $f(t) = 2\sin(t) - \frac{15}{2}t^2 + 2e^{-t} - 6\cos(2t)$

15. $sF(s) - f(0) = as/(s^2 + a^2) - 0 = \mathcal{L}[f'(t)]$

17. $F(s) = 1/(2s) - sT^2/[2(s^2T^2 + \pi^2)]$

Section 4.2

1. $f(t) = (t-2)H(t-2) - (t-2)H(t-3)$

3. $y'' + 3y' + 2y = H(t-1)$

5. $y'' + 4y' + 4y = tH(t-2)$

7. $y'' - 3y' + 2y = e^{-t}H(t-2)$

9. $y'' + y = \sin(t)[1 - H(t-\pi)]$

Section 4.3

1. $F(s) = 2/(s^2 + 2s + 5)$

3. $F(s) = 1/(s-1)^2 + 3/(s^2 - 2s + 10) + (s-2)/(s^2 - 4s + 29)$

5. $F(s) = 2/(s+1)^3 + 2/(s^2 - 2s + 5) + (s+3)/(s^2 + 6s + 18)$

7. $F(s) = e^6 e^{-3s}/(s-2)$

9. $F(s) = 2e^{-s}/s^3 + 2e^{-s}/s^2 + 3e^{-s}/s + e^{-2s}/s$

11. $F(s) = (1 + e^{-s\pi})/(s^2 + 1)$

13. $F(s) = 4(s+3)/(s^2 + 6s + 13)^2$

15. $f(t) = \frac{1}{2}t^2e^{-2t} - \frac{1}{3}t^3e^{-2t}$

17. $f(t) = e^{-t}\cos(t) + 2e^{-t}\sin(t)$

19. $f(t) = e^{-2t} - 2te^{-2t} + \cos(t)e^{-t} + \sin(t)e^{-t}$

21. $f(t) = e^{t-3}H(t-3)$

23. $f(t) = e^{-(t-1)}[\cos(t-1) - \sin(t-1)]H(t-1)$

25. $f(t) = \cos[2(t-1)]H(t-1) + \frac{1}{6}(t-3)^3e^{2(t-3)}H(t-3)$

27. $f(t) = \{\cos[2(t-1)] + \frac{1}{2}\sin[2(t-1)]\}H(t-1) + \frac{1}{6}(t-3)^3H(t-3)$

29. $f(t) = t[H(t) - H(t-a)]$; $F(s) = 1/s^2 - e^{-as}/s^2 - ae^{-as}/s$

31. $F(s) = 1/s^2 - e^{-s}/s^2 - e^{-2s}/s$

33. $F(s) = e^{-s}/s^2 - e^{-2s}/s^2 - e^{-3s}/s$

35. $Y(s) = s/(s^2+4) + 3e^{-4s}/[s(s^2+4)]$

37. $Y(s) = e^{-(s-1)}/[(s-1)(s+1)(s+2)]$

39. $Y(s) = 5s/[(s-1)(s-2)] + e^{-s}/[s^3(s-1)(s-2)]$
$+2e^{-s}/[s^2(s-1)(s-2)] + e^{-s}/[s(s-1)(s-2)]$

41. $Y(s) = 1/[s^2(s+2)(s+1)] + ae^{-as}/[(s+1)^2(s+2)]$
$-e^{-as}/[s^2(s+1)(s+2)] - e^{-as}/[s(s+1)(s+2)]$

43. $\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} s^2/(s^2+a^2) = 1 = f(0)$.

45. $\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} 3s/(s^2-2s+10) = 0 = f(0)$.

47. Yes

49. No

51. No

Section 4.4

1. $F(s) = \dfrac{1}{s^2+1}\coth\left(\dfrac{s\pi}{2}\right)$

3. $F(s) = \dfrac{1-(1+as)e^{-as}}{s^2(1-e^{-2as})}$

Section 4.5

1. $f(t) = e^{-t} - e^{-2t}$

3. $f(t) = \frac{5}{4}e^{-t} - \frac{6}{5}e^{-2t} - \frac{1}{20}e^{3t}$

5. $f(t) = e^{-2t}\cos\left(t + \frac{3\pi}{2}\right)$

7. $f(t) = 2.3584\cos(4t + 0.5586)$

9. $f(t) = \frac{1}{2} + \frac{\sqrt{2}}{2}\cos\left(2t + \frac{5\pi}{4}\right)$

Section 4.6

1.
$$\mathcal{L}(t) = \frac{1}{s^2} = \mathcal{L}(1)\mathcal{L}(1)$$

3.
$$\mathcal{L}\left(e^t - 1\right) = \frac{1}{s-1} - \frac{1}{s} = \frac{1}{s(s-1)} = \mathcal{L}(1)\mathcal{L}(e^t)$$

5.
$$\mathcal{L}[t - \sin(t)] = \frac{1}{s^2} - \frac{1}{s^2+1} = \frac{1}{s^2(s^2+1)} = \mathcal{L}(t)\mathcal{L}[\sin(t)]$$

7.

$$\mathcal{L}\left\{\frac{t^2}{a}-\frac{2}{a^3}[1-\cos(at)]\right\}=\frac{2}{s^3}\left(\frac{a}{s^2+a^2}\right)=\mathcal{L}(t^2)\mathcal{L}[\sin(at)]$$

9.

$$H(t-b)*H(t-a)=\int_a^t H(t-b-x)\,dx=-\int_{t-b-a}^{-b} H(\eta)\,d\eta,$$

if $t > a$ and $\eta = t-b-x$.

11.

$$f(t)=e^t-t-1$$

13. Assuming that $a, b > 0$,

$$\int_0^t \delta(t-x-a)\delta(x-b)\,dx=\delta(t-b-a)$$

Section 4.7

1. $f(t)=1+2t$
3. $f(t)=t+\frac{1}{2}t^2$
5. $f(t)=t^3+\frac{1}{20}t^5$
7. $f(t)=t^2-\frac{1}{3}t^4$
9. $f(t)=5e^{2t}-4e^t-2te^t$
11. $f(t)=(1-t)^2e^{-t}$
13. $f(t)=4+\frac{5}{2}t^2+\frac{1}{24}t^4$
15. $x(t)=2A\sqrt{t}/(\pi C)-Bt/(2C)$

17.

$$f(t)=\frac{\alpha}{\beta^2}\left(e^{\beta^2 t}-1+\frac{2\beta\sqrt{t}}{\sqrt{\pi}}-\frac{2e^{\beta^2 t}}{\sqrt{\pi}}\int_0^{\beta\sqrt{t}} e^{-u^2}\,du\right)$$

Section 4.8

1. $y(t)=\frac{5}{4}e^{2t}-\frac{1}{4}+\frac{1}{2}t$
3. $y(t)=e^{3t}-e^{2t}$
5. $y(t)=-\frac{3}{4}e^{-3t}+\frac{7}{4}e^{-t}+\frac{1}{2}te^{-t}$
7. $y(t)=\frac{3}{4}e^{-t}+\frac{1}{8}e^{t}-\frac{7}{8}e^{-3t}$
9. $y(t)=(t-1)H(t-1)$
11. $y(t)=e^{2t}-e^t+\left[\frac{1}{2}+\frac{1}{2}e^{2(t-1)}-e^{t-1}\right]H(t-1)$

13. $y(t) = \left[1 - e^{-2(t-2)} - 2(t-2)e^{-2(t-2)}\right] H(t-2)$

15. $y(t) = \left[\frac{1}{3}e^{2(t-2)} - \frac{1}{2}e^{t-2} + \frac{1}{6}e^{-(t-2)}\right] H(t-2)$

17. $y(t) = 1 - \cos(t) - [1 - \cos(t-T)]\, H(t-T)$

19.
$$\begin{aligned} y(t) &= e^{-t} - \tfrac{1}{4}e^{-2t} - \tfrac{3}{4} + \tfrac{1}{2}t \\ &\quad - \left[e^{-(t-a)} - \tfrac{1}{4}e^{-2(t-a)} - \tfrac{3}{4} + \tfrac{1}{2}(t-a)\right] H(t-a) \\ &\quad + a\left[\tfrac{1}{2}e^{-2(t-a)} + (t-a)e^{-(t-a)} - \tfrac{1}{2}\right] H(t-a). \end{aligned}$$

21. $y(t) = te^t + 3(t-2)e^{t-2}H(t-2)$

23. $y(t) = 3\left[e^{-2(t-2)} - e^{-3(t-2)}\right] H(t-2)$
$\quad + 4\left[e^{-3(t-5)} - e^{-2(t-5)}\right] H(t-5)$

25. $x(t) = 2e^{t/2} - 2 - t;\ y(t) = e^{t/2} - 1 - t$

27. $x(t) = \frac{1}{2}e^{-t} + \frac{3}{2}e^{-3t};\ y(t) = e^{-t} - 1$

Section 4.9

1. $G(s) = 1/(s+k)$ $\quad g(t) = e^{-kt}$
$\quad a(t) = \left(1 - e^{-kt}\right)/k$

3. $G(s) = 1/(s^2 + 4s + 3)$ $\quad g(t) = \frac{1}{2}\left(e^{-t} - e^{-3t}\right)$
$\quad a(t) = \frac{1}{6}e^{-3t} - \frac{1}{2}e^{-t} + \frac{1}{3}$

5. $G(s) = 1/[(s-2)(s-1)]$ $\quad g(t) = e^{2t} - e^t$
$\quad a(t) = \frac{1}{2} + \frac{1}{2}e^{2t} - e^t$

7. $G(s) = 1/(s^2 - 9)$ $\quad g(t) = \frac{1}{6}\left(e^{3t} - e^{-3t}\right)$
$\quad a(t) = \frac{1}{18}\left(e^{3t} + e^{-3t} - 2\right)$

9. $G(s) = 1/[s(s-1)]$ $\quad g(t) = e^t - 1$
$\quad a(t) = e^t - t - 1$

Section 4.10

1. $f(t) = (2 - t)e^{-2t} - 2e^{-3t}$

3. $f(t) = \left(\frac{1}{4}t^2 - \frac{1}{4}t + \frac{1}{8}\right) e^{2t} - \frac{1}{8}$

5. $f(t) = \left[\frac{1}{2}(t-1) - \frac{1}{4} + \frac{1}{4}e^{-2(t-1)}\right] H(t-1)$

7.

$$f(t) = \frac{e^{-bt}}{\cosh(ab)} - 8ab\sum_{n=1}^{\infty}(-1)^n \frac{\sin[(2n-1)\pi t/(2a)]}{4a^2b^2 + (2n-1)^2\pi^2} + 4\sum_{n=1}^{\infty}(-1)^n \frac{(2n-1)\pi\cos[(2n-1)\pi t/(2a)]}{4a^2b^2 + (2n-1)^2\pi^2}.$$

Section 5.1

1. $F(z) = 2z/(2z-1)$ if $|z| > 1/2$.

3. $F(z) = (z^6 - 1)/(z^6 - z^5)$ if $|z| > 0$.

5. $F(z) = (a^2 + a - z)/[z(z-a)]$ if $|z| > a$.

Section 5.2

1. $F(z) = zTe^{aT}/(ze^{aT} - 1)^2$ 3. $F(z) = z(z+a)/(z-a)^3$

5. $F(z) = [z - \cos(1)]/\{z[z^2 - 2z\cos(1) + 1]\}$

7. $F(z) = z[z\sin(\theta) + \sin(\omega_0 T - \theta)]/[z^2 - 2z\cos(\omega_0 T) + 1]$

9. $F(z) = z/(z+1)$ 11. $f_n * g_n = n + 1$ 13. $f_n * g_n = 2^n/n!$

Section 5.3

1. $f_0 = 0.007143$, $f_1 = 0.08503$, $f_2 = 0.1626$, $f_3 = 0.2328$

3. $f_0 = 0.09836$, $f_1 = 0.3345$, $f_2 = 0.6099$, $f_3 = 0.7935$

5. $f_n = 8 - 8\left(\frac{1}{2}\right)^n - 6n\left(\frac{1}{2}\right)^n$ 7. $f_n = (1 - \alpha^{n+1})/(1 - \alpha)$

9. $f_n = \left(\frac{1}{2}\right)^{n-10} H_{n-10} + \left(\frac{1}{2}\right)^{n-11} H_{n-11}$

11. $f_n = \frac{1}{9}(6n - 4)(-1)^n + \frac{4}{9}\left(\frac{1}{2}\right)^n$ 13. $f_n = a^n/(n!)$

Section 5.4

1. $y_n = 1 + \frac{1}{6}n(n-1)(2n-1)$ 3. $y_n = \frac{1}{2}n(n-1)$

5. $y_n = \frac{1}{6}\left[5^n - (-1)^n\right]$ 7. $y_n = (2n-1)\left(\frac{1}{2}\right)^n + \left(-\frac{1}{2}\right)^n$

9. $y_n = 2^n - n - 1.$ 11. $x_n = 2 + (-1)^n; y_n = 1 + (-1)^n$

13. $x_n = 1 - 2(-6)^n; y_n = -7(-6)^n$

Section 5.5

1. marginally stable 3. unstable

Section 6.1

1. $\lambda_n = (2n-1)^2\pi^2/(4L^2)$ with $y_n(x) = \cos\left[(2n-1)\pi x/(2L)\right]$

3. $\lambda_0 = -1$, $y_0(x) = e^{-x}$ and $\lambda_n = n^2$, $y_n(x) = \sin(nx) - n\cos(nx)$

5. $\lambda_n = -n^4\pi^4/L^4$, $y_n(x) = \sin(n\pi x/L)$

7. $\lambda_n = k_n^2$, $y_n(x) = \sin(k_n x)$ with $k_n = -\tan(k_n)$

9. $\lambda_0 = -m_0^2$, $y_0(x) = \sinh(m_0 x) - m_0\cosh(m_0 x)$ with $\coth(m_0\pi) = m_0$ and $\lambda_n = k_n^2$, $y_n(x) = \sin(k_n x) - k_n\cos(k_n x)$ with $k_n = -\cot(k_n\pi)$

11.

(a) $\lambda_n = n^2\pi^2, \quad y_n(x) = \sin[n\pi\ln(x)]$

(b) $\lambda_n = (2n-1)^2\pi^2/4, \quad y_n(x) = \sin[(2n-1)\pi\ln(x)/2]$

(c) $\lambda_0 = 0, \quad y_0(x) = 1; \qquad \lambda_n = n^2\pi^2, \quad y_n(x) = \cos[n\pi\ln(x)]$

13. $\lambda_n = n^2 + 1$, $y_n(x) = x^{-1}\sin[n\ln(x)]$

Section 6.3

1.

$$f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\left(\frac{n\pi x}{L}\right)$$

3.

$$f(x) = \frac{8L}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^2}\sin\left[\frac{(2n-1)\pi x}{2L}\right]$$

Section 6.4

1.
$$f(x) = \tfrac{1}{4}P_0(x) + \tfrac{1}{2}P_1(x) + \tfrac{5}{16}P_2(x) + \cdots$$

3.
$$f(x) = \tfrac{1}{2}P_0(x) + \tfrac{5}{8}P_2(x) - \tfrac{3}{16}P_4(x) + \cdots$$

5.
$$f(x) = \tfrac{3}{2}P_1(x) - \tfrac{7}{8}P_3(x) + \tfrac{11}{16}P_5(x) + \cdots$$

Section 7.3

1.
$$u(x,t) = \frac{4L}{c\pi^2}\sum_{m=1}^{\infty}\frac{1}{(2m-1)^2}\sin\left[\frac{(2m-1)\pi x}{L}\right]\sin\left[\frac{(2m-1)\pi ct}{L}\right]$$

3.
$$u(x,t) = \frac{9h}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\left(\frac{2n\pi}{3}\right)\sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi ct}{L}\right)$$

5.
$$\begin{aligned} u(x,t) = {} & \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi ct}{L}\right) \\ & + \frac{4aL}{\pi^2 c}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^2}\sin\left[\frac{(2n-1)\pi}{4}\right]\sin\left[\frac{(2n-1)\pi x}{L}\right] \\ & \times \sin\left[\frac{(2n-1)\pi ct}{L}\right] \end{aligned}$$

7.
$$u(x,t) = \frac{4L}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^2}\sin\left[\frac{(2n-1)\pi x}{L}\right]\cos\left[\frac{(2n-1)\pi ct}{L}\right]$$

9.
$$\begin{aligned} u(x,t) = {} & \frac{8L}{\pi^2}e^{-ht}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^2}\sin\left[\frac{(2n-1)\pi x}{2L}\right]\left\{\cos\left[t\sqrt{\lambda_n c^2 - h^2}\right]\right. \\ & \left. + h\sin\left[t\sqrt{\lambda_n c^2 - h^2}\right]\Big/\sqrt{\lambda_n c^2 - h^2}\right\}, \end{aligned}$$

where $\lambda_n = (2n-1)^2\pi^2/4L^2$.

Section 7.4

1.

$$u(x,t) = \sin(2x)\cos(2ct) + \cos(x)\sin(ct)/c$$

3.

$$u(x,t) = \frac{1+x^2+c^2t^2}{(1+x^2+c^2t^2)^2+4x^2c^2t^2} + \frac{e^x\sinh(ct)}{c}$$

5.

$$u(x,t) = \cos\left(\frac{\pi x}{2}\right)\cos\left(\frac{\pi ct}{2}\right) + \frac{\sinh(ax)\sinh(act)}{ac}$$

Section 7.5

1.

$$u(x,t) = \frac{4}{\pi^2}\sum_{m=1}^{\infty}\frac{\sin[(2m-1)\pi x]\sin[(2m-1)\pi t]}{(2m-1)^2}$$

3.

$$u(x,t) = \sin(\pi x)\cos(\pi t) - \sin(\pi x)\sin(\pi t)/\pi$$

5.

$$u(x,t) = xt - te^{-x} + \sinh(t)e^{-x} + \left[1 - e^{-(t-x)} + t - x - \sinh(t-x)\right]H(t-x)$$

7.

$$u(x,t) = \frac{gx}{\omega^2} - \frac{2g\omega^2}{L}\sum_{n=1}^{\infty}\frac{\sin(\lambda_n x)\cos(\lambda_n t)}{\lambda_n^2(\omega^4+\omega^2/L+\lambda_n^2)\sin(\lambda_n L)},$$

where λ_n is the nth root of $\lambda = \omega^2\cot(\lambda L)$.

9.

$$u(x,t) = E - \frac{4E}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin\left[\frac{(2n-1)\pi x}{2\ell}\right]\cos\left[\frac{(2n-1)c\pi t}{2\ell}\right]$$

or

$$u(x,t) = E\sum_{n=0}^{\infty}(-1)^n H\left(t - \frac{x+2n\ell}{c}\right) + E\sum_{n=0}^{\infty}(-1)^n H\left\{t - \frac{[(2n+2)\ell - x]}{c}\right\}$$

11.

$$p(x,t) = p_0 - \frac{4\rho u_0 c}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}\sin\left[\frac{(2n-1)\pi x}{2L}\right]\sin\left[\frac{(2n-1)c\pi t}{2L}\right]$$

13.

$$u(x,t) = \frac{gt^2}{2} - \frac{gL^2}{6c^2} - \frac{2gL^2}{c^2\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi ct}{L}\right)$$

Section 8.3

1.

$$u(x,t) = \frac{4A}{\pi}\sum_{m=1}^{\infty}\frac{\sin[(2m-1)x]}{2m-1}e^{-a^2(2m-1)^2 t}$$

3.

$$u(x,t) = -2\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin(nx)e^{-a^2n^2t}$$

5.

$$u(x,t) = \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{(2m-1)^2}\sin[(2m-1)x]e^{-a^2(2m-1)^2t}$$

7.

$$u(x,t) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{\cos[(2m-1)x]}{(2m-1)^2}e^{-a^2(2m-1)^2t}$$

9.

$$u(x,t) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos[(2n-1)x]}{(2n-1)^2}e^{-a^2(2n-1)^2t}$$

11.

$$u(x,t) = \frac{32}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^3} \cos\left[\frac{(2n-1)x}{2}\right] e^{-a^2(2n-1)^2 t/4}$$

13.

$$u(x,t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin[(2n-1)x/2]}{2n-1} e^{-a^2(2n-1)^2 t/4}$$

15.

$$u(x,t) = \sum_{n=1}^{\infty} \left[\frac{4}{2n-1} - \frac{8(-1)^{n+1}}{(2n-1)^2\pi}\right] \sin\left[\frac{(2n-1)x}{2}\right] e^{-a^2(2n-1)^2 t/4}$$

17.

$$u(x,t) = \frac{T_0 x}{\pi} + \frac{2T_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin(nx) e^{-a^2 n^2 t}$$

19.

$$u(x,t) = h_1 + \frac{(h_2 - h_1)x}{L} + \frac{2(h_2 - h_1)}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin\left(\frac{n\pi x}{L}\right) \exp\left(-\frac{a^2 n^2 \pi^2}{L^2} t\right)$$

21.

$$u(x,t) = h_0 - \frac{4h_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin\left[\frac{(2n-1)\pi x}{L}\right] \exp\left[-\frac{(2n-1)^2\pi^2 a^2 t}{L^2}\right]$$

23.

$$u(x,t) = \frac{1}{3} - t - \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos(n\pi x) e^{-a^2 n^2 \pi^2 t}$$

25.

$$u(x,t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^4} \sin[(2n-1)x] \left[1 - e^{-(2n-1)^2 t}\right]$$

27.

$$u(x,t) = \frac{A_0(L^2 - x^2)}{2\kappa} + \frac{A_0 L}{h}$$
$$- \frac{2L^2 A_0}{\kappa} \sum_{n=1}^{\infty} \frac{\sin(\beta_n)}{\beta_n^4[1 + \kappa \sin^2(\kappa)/hL]} \cos\left(\frac{\beta_n x}{L}\right) \exp\left(-\frac{a^2 \beta_n^2 t}{L^2}\right),$$

where β_n is the nth root of $\beta \tan(\beta) = \kappa/hL$.

29.

$$u(r,t) = \frac{2}{\pi r} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(n\pi r) e^{-a^2 n^2 \pi^2 t}$$

31.

$$u(r,t) = \frac{G}{4\rho\nu}(b^2 - r^2) - \frac{2Gb^2}{\rho\nu} \sum_{n=1}^{\infty} \frac{J_0(k_n r/b)}{k_n^3 J_1(k_n)} \exp\left(-\frac{\nu k_n^2}{b^2} t\right),$$

where k_n is the nth root of $J_0(k) = 0$.

Section 8.4

1.

$$u(x,t) = T_0\left(1 - e^{-a^2 t}\right)$$

3.

$$u(x,t) = x + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin(n\pi x) e^{-n^2 \pi^2 t}$$

5.

$$u(x,t) = \frac{x(1-x)}{2} - \frac{4}{\pi^3} \sum_{m=1}^{\infty} \frac{\sin[(2m-1)\pi x]}{(2m-1)^3} e^{-(2m-1)^2 \pi^2 t}$$

7.

$$u(x,t) = x \operatorname{erfc}\left(\frac{x}{2\sqrt{t}}\right) - 2\sqrt{\frac{t}{\pi}} \exp\left(-\frac{x^2}{4t}\right)$$

9.

$$u(x,t) = \frac{u_0}{2} e^{-\delta x} \operatorname{erfc}\left(\frac{x}{2a\sqrt{t}} + \frac{a(1-\delta)\sqrt{t}}{2}\right)$$
$$+ \frac{u_0}{2} e^{-x} \operatorname{erfc}\left(\frac{x}{2a\sqrt{t}} - \frac{a(1-\delta)\sqrt{t}}{2}\right)$$

11.

$$u(x,t) = \frac{t(L-x)}{L} + \frac{Px(x-L)}{2a^2} - \frac{x(x-L)(x-2L)}{6a^2L} - \frac{2PL^2}{a^2\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}\sin\left(\frac{n\pi x}{L}\right)\exp\left(-\frac{a^2n^2\pi^2t}{L^2}\right) + \frac{2(P+1)L^2}{a^2\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^3}\sin\left(\frac{n\pi x}{L}\right)\exp\left(-\frac{a^2n^2\pi^2t}{L^2}\right).$$

13.

$$u(r,t) = \frac{r^2}{2} + 3t - \frac{3}{10} - \frac{2}{r}\sum_{n=1}^{\infty}\frac{\sin(\lambda_n r)}{\lambda_n^2\sin(\lambda_n)}e^{-\lambda_n^2 t},$$

where $\tan(\lambda_n) = \lambda_n$.

15.

$$y(t) = \frac{4\mu A\omega^2}{mL}\sum_{n=1}^{\infty}\frac{\lambda_n e^{\lambda_n t}}{\lambda_n^4 - (\frac{2\mu}{mL})(1+\frac{2\mu L}{m\nu})\lambda_n^3 + 2\omega^2\lambda_n^2 + \frac{6\omega^2\mu}{mL}\lambda_n + \omega^4},$$

where λ_n is the nth root of $\lambda^2 + 2\mu\lambda^{3/2}\coth(L\sqrt{\lambda/\nu})/(m\sqrt{\nu}) + \omega^2 = 0$.

17.

$$u(x,t) = 1 - 2e^{Vx/2 - V^2t/4} \times \sum_{n=1}^{\infty}\frac{\lambda_n\{(V/2)\sin[\lambda_n(1-x)] + \lambda_n\cos[\lambda_n(1-x)]\}e^{-\lambda_n^2 t}}{(\lambda_n^2 + V^2/4)[\lambda_n\sin(\lambda_n) - (1+V/2)\cos(\lambda_n)]},$$

where λ_n is the nth root of $\lambda\cot(\lambda) = -V/2$.

19.

$$u(r,t) = \frac{a^2 - r^2}{4} - 2a^2\sum_{n=1}^{\infty}\frac{J_0(k_n r/a)}{k_n^3 J_1(k_n)}e^{-k_n^2 t/a^2},$$

where k_n is the nth root of $J_0(k) = 0$.

Section 8.5

1.

$$u(x,t) = \tfrac{1}{2}\text{erf}\left(\frac{b-x}{\sqrt{4a^2t}}\right) + \tfrac{1}{2}\text{erf}\left(\frac{b+x}{\sqrt{4a^2t}}\right)$$

3.

$$u(x,t) = \tfrac{1}{2}T_0 \text{erf}\left(\frac{b-x}{\sqrt{4a^2t}}\right) + \tfrac{1}{2}T_0 \text{erf}\left(\frac{x}{\sqrt{4a^2t}}\right).$$

Section 8.6

1.

$$u(x,t) = \frac{4a^2\pi}{L^2}\sum_{m=1}^{\infty}(2m-1)\sin\left[\frac{(2m-1)\pi x}{L}\right] e^{-a^2(2m-1)^2\pi^2 t/L^2} \times \int_0^t f(\tau)e^{a^2(2m-1)^2\pi^2\tau/L^2}\,d\tau$$

3.

$$u(x,t) = \frac{2}{\sqrt{\pi}}\int_{x/\sqrt{4\nu t}}^{\infty} V\left(t - \frac{x^2}{4\nu\eta^2}\right) e^{-\eta^2}\,d\eta$$

Section 9.3

1.

$$u(x,y) = \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{\sinh[(2m-1)\pi(a-x)/b]\sin[(2m-1)\pi y/b]}{(2m-1)\sinh[(2m-1)\pi a/b]}.$$

3.

$$u(x,y) = -\frac{2a}{\pi}\sum_{n=1}^{\infty}\frac{\sinh(n\pi y/a)\sin(n\pi x/a)}{n\ \sinh(n\pi b/a)}$$

5.

$$u(x,y) = \frac{4}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sinh[(2n-1)\pi y/2a]\cos[(2n-1)\pi x/2a]}{(2n-1)\sinh[(2n-1)\pi b/2a]}$$

7.

$$u(x,y) = 1$$

9.

$$u(x,y) = 1 - \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{\cosh[(2m-1)\pi y/a]\sin[(2m-1)\pi x/a]}{(2m-1)\cosh[(2m-1)\pi b/a]}$$

11.

$$u(x, y) = 1$$

13.

$$u(x, y) = T_0 + \Delta T \cos(2\pi x/\lambda)e^{-2\pi y/\lambda}$$

15.

$$u(r, z) = 2a \sum_{n=1}^{\infty} \frac{\sinh(k_n z/a)\, J_0(k_n r/a)}{k_n^2 \cosh(k_n L/a) J_1(k_n)},$$

where k_n is the nth root of $J_0(k) = 0$.

17.

$$u(r, z) = \frac{2}{b^2 - a^2} \sum_{n=1}^{\infty} \frac{[bJ_1(k_n b) - aJ_1(k_n a)] J_0(k_n r) \cosh(k_n z)}{k_n \cosh(k_n d) J_0^2(k_n)},$$

where k_n is the nth root of $J_1(k) = 0$.

19.

$$u(r, z) = -\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n I_1(n\pi r) \sin(n\pi z)}{n\, I_1(n\pi a)}$$

21.

$$u(r, z) = 2B \sum_{n=1}^{\infty} \frac{\exp[z(1 - \sqrt{1 + 4k_n^2}\,)/2] J_0\,(k_n r)}{(k_n^2 + B^2) J_0(k_n)},$$

where k_n is the nth root of $k\, J_1(k) = B\, J_0(k)$.

23.

$$u(r, \theta) = 50 \sum_{m=1}^{\infty} [P_{2m-2}(0) - P_{2m}(0)] \left(\frac{r}{a}\right)^{2m-1} P_{2m-1}[\cos(\theta)]$$

25. T_0

Section 9.4

1.

$$u(x, y) = \frac{1}{\pi}\left[\tan^{-1}\left(\frac{1 - x}{y}\right) + \tan^{-1}\left(\frac{x}{y}\right)\right]$$

3.

$$u(x,y) = \frac{T_0}{\pi}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{x}{y}\right)\right]$$

5.

$$\begin{aligned} u(x,y) = {} & \frac{T_0}{\pi}\left[\tan^{-1}\left(\frac{1-x}{y}\right) + \tan^{-1}\left(\frac{1+x}{y}\right)\right] \\ & + \frac{T_1 - T_0}{2\pi}\, y \ln\left[\frac{(x-1)^2+y^2}{x^2+y^2}\right] \\ & + \frac{T_1 - T_0}{\pi}\, x \left[\tan^{-1}\left(\frac{1-x}{y}\right) + \tan^{-1}\left(\frac{x}{y}\right)\right] \end{aligned}$$

Section 9.5

1.

$$\begin{aligned} u(x,y) = {} & \frac{64\,R}{\pi^4 T}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^{n+1}(-1)^{m+1}}{(2n-1)(2m-1)} \\ & \times \frac{\cos[(2n-1)\pi x/2a]\cos[(2m-1)\pi y/b]}{(2n-1)(2m-1)[(2n-1)^2/a^2 + (2m-1)^2/b^2]} \end{aligned}$$

Section 9.6

1.

$$u(x,y) = \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{1}{2m-1}\exp\left[-\frac{(2m-1)\pi x}{a}\right]\sin\left[\frac{(2m-1)\pi y}{a}\right]$$

Section 10.1

1. $\mathbf{a}\times\mathbf{b} = -3\mathbf{i} + 19\mathbf{j} + 10\mathbf{k}$

3. $\mathbf{a}\times\mathbf{b} = \mathbf{i} - 8\mathbf{j} + 7\mathbf{k}$

5. $\mathbf{a}\times\mathbf{b} = -3\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$

7.

$$\begin{aligned} \mathbf{a}\times(\mathbf{b}\times\mathbf{c}) + \mathbf{b}\times(\mathbf{c}\times\mathbf{a}) + \mathbf{c}\times(\mathbf{a}\times\mathbf{b}) &= (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c} + (\mathbf{b}\cdot\mathbf{a})\mathbf{c} \\ &\quad - (\mathbf{b}\cdot\mathbf{c})\mathbf{a} + (\mathbf{c}\cdot\mathbf{b})\mathbf{a} - (\mathbf{c}\cdot\mathbf{a})\mathbf{b} \\ &= 0 \end{aligned}$$

9.

$$\nabla f = y\cos(yz)\mathbf{i} + [x\cos(yz) - xyz\sin(yz)]\mathbf{j} - xy^2\sin(yz)\mathbf{k}$$

11.

$$\nabla f = 2xy^2(2z+1)^2\mathbf{i} + 2x^2y(2z+1)^2\mathbf{j} + 4x^2y^2(2z+1)\mathbf{k}$$

13. Plane parallel to the xy plane at height of $z = 3$, $\mathbf{n} = \mathbf{k}$

15. Paraboloid,

$$\mathbf{n} = -\frac{2x}{\sqrt{1+4x^2+4y^2}}\mathbf{i} - \frac{2y}{\sqrt{1+4x^2+4y^2}}\mathbf{j} + \frac{1}{\sqrt{1+4x^2+4y^2}}\mathbf{k}$$

17. A plane, $\mathbf{n} = \mathbf{j}/\sqrt{2} - \mathbf{k}/\sqrt{2}$

19. A parabola of infinite extent along the y-axis, $\mathbf{n} = -2x\mathbf{i}/\sqrt{1+4x^2} + \mathbf{k}/\sqrt{1+4x^2}$

21. $y = 2/(x+1); \quad z = \exp[(y-1)/y]$

23. $y = x; \quad z^2 = y/(3y-2)$

Section 10.2

1.

$$\nabla \cdot \mathbf{F} = 2xz + z^2$$
$$\nabla \times \mathbf{F} = (2xy - 2yz)\mathbf{i} + (x^2 - y^2)\mathbf{j}$$
$$\nabla(\nabla \cdot \mathbf{F}) = 2z\mathbf{i} + (2x + 2z)\mathbf{k}$$

3.

$$\nabla \cdot \mathbf{F} = 2(x-y) - xe^{-xy} + xe^{2y}$$
$$\nabla \times \mathbf{F} = 2xze^{2y}\mathbf{i} - ze^{2y}\mathbf{j} + \left[2(x-y) - ye^{-xy}\right]\mathbf{k}$$
$$\nabla(\nabla \cdot \mathbf{F}) = \left(2 - e^{-xy} + xye^{-xy} + e^{2y}\right)\mathbf{i} + \left(x^2e^{-xy} + 2xe^{2y} - 2\right)\mathbf{j}$$

5.

$$\nabla \cdot \mathbf{F} = 0$$
$$\nabla \times \mathbf{F} = -x^2\mathbf{i} + (5y - 9x^2)\mathbf{j} + (2xz - 5z)\mathbf{k}$$
$$\nabla(\nabla \cdot \mathbf{F}) = \mathbf{0}$$

7.

$$\nabla \cdot \mathbf{F} = e^{-y} + z^2 - 3e^{-z}$$

$$\nabla \times \mathbf{F} = -2yz\mathbf{i} + xe^{-y}\mathbf{k}$$

$$\nabla(\nabla \cdot \mathbf{F}) = -e^{-y}\mathbf{j} + (2z + 3e^{-z})\mathbf{k}$$

9.

$$\nabla \cdot \mathbf{F} = yz + x^3ze^z + xye^z$$

$$\nabla \times \mathbf{F} = (xe^z - x^3ye^z - x^3yze^z)\mathbf{i} + (xy - ye^z)\mathbf{j} + (3x^2yze^z - xz)\mathbf{k}$$

$$\nabla(\nabla \cdot \mathbf{F}) = \left(3x^2ze^z + ye^z\right)\mathbf{i} + \left(z + xe^z\right)\mathbf{j} + \left(y + x^3e^z + x^3ze^z + xye^z\right)\mathbf{k}$$

11.

$$\nabla \cdot \mathbf{F} = y^2 + xz^2 - xy\sin(z)$$

$$\nabla \times \mathbf{F} = [x\cos(z) - 2xyz]\mathbf{i} - y\cos(z)\mathbf{j} + (yz^2 - 2xy)\mathbf{k}$$

$$\nabla(\nabla \cdot \mathbf{F}) = [z^2 - y\sin(z)]\mathbf{i} + \left[2y - x\sin(z)\right]\mathbf{j} + \left[2xz - xy\cos(z)\right]\mathbf{k}$$

13.

$$\nabla \cdot \mathbf{F} = y^2 + xz - xy\sin(z)$$

$$\nabla \times \mathbf{F} = [x\cos(z) - xy]\mathbf{i} - y\cos(z)\mathbf{j} + (yz - 2xy)\mathbf{k}$$

$$\nabla(\nabla \cdot \mathbf{F}) = [z - y\sin(z)]\mathbf{i} + \left[2y - x\sin(z)\right]\mathbf{j} + \left[x - xy\cos(z)\right]\mathbf{k}$$

Section 10.3

1. $16/7 + 2/(3\pi)$ 3. $e^2 + 2e^8/3 + e^{64}/2 - 13/6$ 5. -4π

7. 0 9. 2π

Section 10.4

1. $\varphi(x, y, z) = x^2y + y^2z + 4z + \text{constant}$

3. $\varphi(x, y, z) = xyz + \text{constant}$

5. $\varphi(x, y, z) = x^2\sin(y) + xe^{3z} + 4z + \text{constant}$

7. $\varphi(x, y, z) = xe^{2z} + y^3 + \text{constant}$

9. $\varphi(x, y, z) = xy + xz + \text{constant}$

Section 10.5

1. 1/2
3. 0
5. 27/2
7. 5
9. 0
11. 40/3
13. 86/3
15. 96π

Section 10.6

1. -5
3. 1
5. 0
7. 0
9. -16π
11. -2

Section 10.7

1. -10
3. 2
5. π
7. 45/2

Section 10.8

1. 3
3. -16
5. 4π
7. 5/12

Section 11.1

1.
$$A + B = \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix} = B + A$$

3.
$$3A - 2B = \begin{pmatrix} 7 & 10 \\ -1 & 2 \end{pmatrix}, \qquad 3(2A - B) = \begin{pmatrix} 15 & 21 \\ 0 & 6 \end{pmatrix}$$

5.
$$(A + B)^T = \begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix}, \qquad A^T + B^T = \begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix}$$

7.
$$AB = \begin{pmatrix} 11 & 11 \\ 5 & 5 \end{pmatrix}, \qquad A^T B = \begin{pmatrix} 5 & 5 \\ 8 & 8 \end{pmatrix}$$
$$BA = \begin{pmatrix} 4 & 6 \\ 8 & 12 \end{pmatrix}, \qquad B^T A = \begin{pmatrix} 5 & 8 \\ 5 & 8 \end{pmatrix}$$

9.
$$BB^T = \begin{pmatrix} 2 & 4 \\ 4 & 8 \end{pmatrix}, \qquad B^T B = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$$

11.

$$A^3 + 2A = \begin{pmatrix} 65 & 100 \\ 25 & 40 \end{pmatrix}$$

13. yes $\begin{pmatrix} 27 & 11 \\ 2 & 5 \end{pmatrix}$ 15. yes $\begin{pmatrix} 11 & 8 \\ 8 & 4 \\ 5 & 3 \end{pmatrix}$ 17. no

19.

$$5(2A) = \begin{pmatrix} 10 & 10 \\ 10 & 20 \\ 30 & 10 \end{pmatrix} = 10A$$

21.

$$(A + B) + C = \begin{pmatrix} 4 & 0 \\ 8 & 2 \end{pmatrix} = A + (B + C)$$

23.

$$A(B + C) = \begin{pmatrix} 9 & -1 \\ 11 & -2 \end{pmatrix} = AB + AC$$

25.

$$\begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

27.

$$\begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

29.

$$\begin{pmatrix} 0 & 1 & 2 & 3 \\ 3 & 0 & -4 & -4 \\ 1 & 1 & 1 & 1 \\ 2 & -3 & 1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ -3 \\ 7 \end{pmatrix}$$

Section 11.2

1. 7 3. 1 5. −24 7. 3

Section 11.3

1. $x_1 = \frac{9}{5}$, $x_2 = \frac{3}{5}$ 3. $x_1 = 0$, $x_2 = 0$, $x_3 = -2$

Section 11.4

1. $x_2 = 2,\ x_1 = 1$

3. $x_3 = \alpha,\ x_2 = -\alpha,\ x_1 = \alpha$

5. $x_3 = \alpha,\ x_2 = 2\alpha,\ x_1 = -1$

7. $x_3 = 2.2,\ x_2 = 2.6,\ x_1 = 1$

9. $A^{-1} = \begin{pmatrix} -1/13 & 5/13 \\ 2/13 & 3/13 \end{pmatrix}$

11. $A^{-1} = \begin{pmatrix} 1 & 2 & 5 \\ 0 & -1 & 2 \\ 2 & 4 & 11 \end{pmatrix}$

Section 11.5

1.
$$\lambda = 4, \quad \mathbf{x}_0 = \alpha \begin{pmatrix} 2 \\ 1 \end{pmatrix}; \qquad \lambda = -3 \quad \mathbf{x}_0 = \beta \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

3.
$$\lambda = 1 \quad \mathbf{x}_0 = \alpha \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}; \qquad \lambda = 0, \quad \mathbf{x}_0 = \gamma \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

5.
$$\lambda = 1, \quad \mathbf{x}_0 = \alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}; \qquad \lambda = 2, \quad \mathbf{x}_0 = \gamma \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

7.
$$\lambda = 0, \quad \mathbf{x}_0 = \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}; \ \lambda = 1, \quad \mathbf{x}_0 = \beta \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}; \ \lambda = 2, \quad \mathbf{x}_0 = \gamma \begin{pmatrix} 7 \\ 3 \\ 1 \end{pmatrix}$$

Section 11.6

1.
$$\mathbf{x} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{3t}.$$

3.
$$\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-t}.$$

5.

$$\mathbf{x} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t/2} + c_2 \begin{pmatrix} t \\ -1/2 - t \end{pmatrix} e^{t/2}.$$

7.

$$\mathbf{x} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} -1 + t \\ -t \end{pmatrix} e^{2t}.$$

9.

$$\mathbf{x} = c_3 \begin{pmatrix} -3\cos(2t) - 2\sin(2t) \\ \cos(2t) \end{pmatrix} e^t + c_4 \begin{pmatrix} 2\cos(2t) - 3\sin(2t) \\ \sin(2t) \end{pmatrix} e^t$$

11.

$$\mathbf{x} = c_3 \begin{pmatrix} 2\cos(t) \\ 7\cos(t) + \sin(t) \end{pmatrix} e^{-3t} + c_4 \begin{pmatrix} 2\sin(t) \\ 7\sin(t) - \cos(t) \end{pmatrix} e^{-3t}$$

13.

$$\mathbf{x} = c_3 \begin{pmatrix} -\cos(2t) + \sin(2t) \\ \cos(2t) \end{pmatrix} e^t + c_4 \begin{pmatrix} -\cos(2t) - \sin(2t) \\ \sin(2t) \end{pmatrix} e^t$$

15.

$$\mathbf{x} = c_1 \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} -3 \\ 2 \end{pmatrix} e^{-t}$$

17.

$$\mathbf{x} = c_1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} e^t + c_3 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} e^{2t}$$

19.

$$\mathbf{x} = c_1 \begin{pmatrix} 3 \\ -2 \\ 12 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} e^t + c_3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{2t}$$